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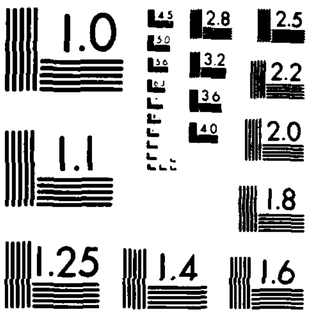

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on

A Study of Texture Analysis Algorithms

by

Charles A. Harlow
Richard W. Conners

April 1981

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This research has focused upon developing improved texture analysis algorithms. Work performed during the second year of the grant has shown that the Spatial Gray Level Dependence (SGLDM) texture analysis algorithm is a superior algorithm under fairly weak assumptions. For this reason our subsequent work has continued the development of the SGLDM method. Tiling theory has been combined with the SGLDM analysis procedure to create a structural (SSA) analyzer for texture patterns. Recent work has focused upon determining measures derived from the SGLDM cooccurence matrices that characterize texture patterns. It has been shown that the commonly used measures are inadequate. A texture generation (CONTINUED)

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ITEM #20, CONTINUED: procedure has been developed and this has been used to generate new measures based upon the perceptual concepts of uniformity and proximity. These measures offer promise of developing measures related to perceptual features. Experiments were also conducted which shows that the SGLDM algorithm can discriminate known counterexamples to the Julesz conjecture. Thus the robustness of the SGLDM has been further established over this theoretically troublesome class of textures.

ABSTRACT

This research has focused upon developing improved texture analysis algorithms. Work performed during the second year of the grant has shown that the Spatial Gray Level Dependence (SGLDM) texture analysis algorithm is a superior algorithm under fairly weak assumptions. For this reason our subsequent work has continued the development of the SGLDM method. Tiling theory has been combined with the SGLDM analysis procedure to create a structural (SSA) analyzer for texture patterns. Recent work has focused upon determining measures derived from the SGLDM cooccurence matrices that characterize texture patterns. It has been shown that the commonly used measures are inadequate. A texture generation procedure has been developed and this has been used to generate new measures based upon the perceptual concepts of uniformity and proximity. These measures offer premise of developing measures related to perceptual features. Experiments were also conducted which shows that the SGLDM algorithm can discriminate known counterexamples to the Julesz conjecture. Thus the robustness of the SGLDM has been further established over this theoretically troublesome class of textures.

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I. INTRODUCTION

This research project is concerned with developing a better understanding of texture analysis algorithms. It is believed that an understanding of texture analysis is essential if one is to build effective image understanding systems. Our approach to developing improved texture analysis methods has been to remove the heuristics so often used and instead rely upon mathematical theory and sound experiments.

During the current year substantial progress has been made in developing our system. Two Perkin Elmer 32 bit computers have been expanded to 1M Bytes of memory each. A substantial number of tape drives and disc drives have also been added. A Television Scanning System and a map digitizing table have also been brought on-line. The vidicon scanning system allows one to scan images at eigher 256 x 256, 512 x 512 or 1024 x 1024 resolution with 8 bits of gray level information. The system includes a computer controlled XY stage to move the images under the camera. The XY state can move a maximum of 16" in either the X or Y direction. Also purchased was a 600 mm - 150 manual zoom lens. This system provides us, for the first time, the capability to scan images inhouse. It will allow us to apply the methodologies we develop to real world data to verify their robustness.

Also a Daedaleus multispectral scanner (12 channels) has been acquired and installed in an aircraft. The airplane system is housed within the Louisiana Department of Natural Resources. This system can digitize 11 channels of data simultaneously. Of these 11 channels ten are fixed as to the wavelengths they measure. These are listed below:

		Wavelength
Channel 1		.38- .42μm
Channel 2		.42- .45μm
Channel 3		.45- .50μm
Channel 4		.50- .55μm
Channel 5		.55- .60μm
Channel 6		.60- .65μm
Channel 7		.65- .69μm
Channel 8		.70- .79μm
Channel 9		.80- .89μm
Channel 10		.92-1 .10μm

The 11th channel can either be set to measure an IR region in the 8. - 14.μm range or a UV region in the .3 - .38μm range. The system is valued at $500,000. This system is unique to universities and should greatly enhance our data collection abilities. Under contract to RADC our software system has been expanded to include a comprehensive texture analysis module. This system will not be reported in detail here but provides very useful facilities for experimenting with and analyzing texture patterns.

Our work on texture analysis has continued the development of the structural analyzer (SSA) for texture patterns. It has also focused upon methodologies which can match the mechanisms of humans perception. The desire has been to create operators which can describe and discriminate visually distinct textures. In this regard new measures have been developed that characterize the visual properties of uniformity and proximity.

II. OBJECTIVES

The objectives of the work during the current year were to:

1. Develop Improved Method for Generating Texture Patterns.

2. Study the Adequacy of the SGLDM Texture Analysis Method to Model Primitive Mechanism of Human Texture Perception.

3. Fully Investigate the Properties of the Unit Cell for Texture Patterns.

4. Further Develop the Results on Using Tiling Theory in Structural Analysis of Textures.

5. Develop New Features for the SGLDM.

6. Develop Methods for Measuring the Similarity of Texture Patterns.

Substantial progress has been obtained in meeting these objectives. The work will be described in the next section.

III. STATUS OF RESEARCH

Our research work has concentrated upon developing the statistical and structural texture analysis system (SSA), verifying the sufficiency of first and second order statistics for texture discrimination and identifying new measures derived from the SGLDM (cooccurence) matrices that characterize visually perceived properties of texture patterns.

A. *The Statistical Structural Analyzer*

The Statistical Structural Analyzer (SSA) represents an attempt by the authors to create a structural textural analyzer based on statistical methods. The SSA was first introduced in [1]. Here it was shown that the SSA could be used to detect periodicity and also could be used to characterize the size and shape of unit patterns and the placement rules of these patterns in periodic and almost periodic textures; things which should be the heart of any structural textural analyzer.

The SSA is formulated around the Spatial Gray Level Dependence Method (SGLDM) [1, 2, 3, 4, 5, 6] for doing texture analysis. Computationally the heart of the SGLDM is the spatial gray level dependence matrices $S(\delta) = [s(i,j,\delta)]$. An element $s(i,j,\delta)$ of the matrix $S(\delta)$ represents the estimated probability of going from gray level i to gray level j given the displacement sector $\delta = (\Delta n, \Delta m)$ where Δn and Δm are integers.

In what follows it is convenient to think of δ in the polar form, $\delta = (d,\theta)$ rather than in the cartesian form $(\Delta n, \Delta m)$. The conversion between the cartesian form and the convenient polar form is given by

$$d = \max\,[\Delta n,\Delta m]$$

$$\theta = \arctan\,(\Delta m/\Delta n).$$

In this polar form the parameter d is referred to as the intersample spacing distance and θ is referred to as the angular orientation.

In using the SGLDM with the SSA, the matrix $S(\delta)$ is not forced to be symmetric as is commonly done by many investigators. The reason for not forcing the matrix $S(\delta)$ to be symmetric is discussed in [2]. Consequently $S(\delta)$ for $\delta = (d,\theta)$ may not equal $S(\delta')$ for $\delta' = (d,\theta+180°)$.

It is of interest to note that if $S(\delta)$ is computed from a random field $X(n,m)$ which is ergodic and translation stationary of order two and further if the region from which $S(\delta)$ is calculated is square with side h then

$$\lim_{h\to\infty} S(\delta) = [P(X(0) = i,\ X(0+\delta) = j)] \qquad (1)$$

where 0 is the zero vector.

During the course of the last funding year we have concentrated on the continued development of the SSA. Further during the proposed funding year it is our intention to continue to develop the SSA. Consequently it seems worthwhile to explain why our emphasis is so concentrated. The best way to do this would seemingly be to review the assumptions behind the SSA and state the scientific evidence supporting each assumption.

The first assumption stated is the most important and fundamental to all our research. It explains our concentration on developing an advanced texture analysis procedure around the SGLDM.

Assumption I: The SGLDM is the most powerful statistical texture analysis algorithm. That is, it is assumed that the intermediate matrices, namely, the spatial gray level dependence matrices, contain more important texture-context information than the intermediate matrices of any other statistical texture analysis algorithm.

Since this is such an important assumption, one in which all of our work depends, it seems advisable to discuss the supporting evidence in some detail. Basically this evidence comes from studies done by perceptual psychologists, comparison studies done evaluating a number of texture analysis algorithms and finally the published successful uses of the SGLDM is solving real world texture analysis problems.

As to the studies done by perceptual psychologists, the most notable of these is the one done by Bela Julesz [7] in 1962. In this study Julesz presented experimental evidence which showed that if two textures can be spontaneously discriminated by the human then these textures have different second-order probabilities. This has become known as the Julesz conjecture. Further he showed that two textures can have different second-order probabilities but still not be discriminable by the human visual system. An example of such a texture pair is given in Figure 1. The texture pair in this example came from [7].

Further experimental conformation of the Julesz conjecture was provided by Pratt et al in a recent paper [8] and also by Julesz, himself, in a 1973 paper [9]. The Julesz conjecture has been the cause of some debate among perceptual psychologists. Pollack [10] and Purks and Richards [11] have on separate occasions claimed to have valid counterexamples. However, these counterexamples are not true ones [8].

Further substantiation of Assumption 1 comes from two comparison studies which have been done which evaluate a number of texture algorithms. These comparison studies are of some interest within themselves so we will digress a bit to discuss them.

Beginning in the late 1960's and continuing up until today a number of texture analysis algorithms have been put forth in the literature. Most if not all of the algorithms measure all or some subset of the second-order probabilities either directly or indirectly, i.e. they measure some function of these probabilities. Consequently few if any of these algorithms can be immediately dismissed as being inferior.

The problem that presents itself is that given a particular texture analysis problem which algorithm would be the best one to use to attack it.

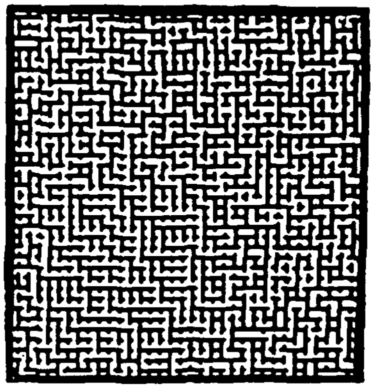


Figure 1. Two textures which have different second-order probabilistics but which are not discriminable to the human observer. The larger field contain (a), (b), (c) and (d) micropatterns of Figure 2 whereas the smaller field only (a) and (c).

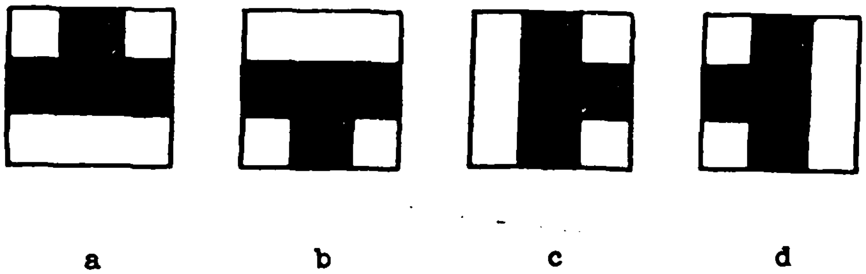


Figure 2. Micropatterns used to generate textures in Figure 1.

The first true comparison study was done by Weszka, Dyer and Rosenfeld [12]. The algorithms compared were the SGLDM; the Gray Level Difference Method (GLDM), [12]; the Power Spectral Method (PSM) [13]; and the Gray Level Run Length Method (GLRLM) [14]. The standard for comparison used in this study was the percentage of overall correct classification obtained on a fixed data base of textural types. The results of the study indicated that the SGLDM and GLDM were about equal in ability and both were better than the PSM or GLRLM.

The next comparison study [2] was performed by the authors and considered the same four algorithms. This comparison method attempted to determine information about these algorithms which cannot be obtained from the classification result comparison method (CRC) used by Weszka, Dyer and Rosenfeld. The difficulty with the CRC is that it compares the whole texture analysis systems. In particular it cannot be used to determine whether the poor performance of the PSM on a particular data base is caused because the information which will allow the discrimination of the textural classes is not contained in the power spectrum or rather because the features defined off the power spectrum do not reflect the differences in the power spectrum which would allow the discrimination. The theoretical comparison employed considered only the amount of texture discrimination information contained in the intermediate matrices of each of the algorithms. The results of this comparison study showed that the SGLDM was the most powerful of the four algorithms. Further counterexamples were given which showed visually distinct textures which could not be discriminated by each of the other three algorithms.

Both of these comparison studies indicate that the SGLDM is a powerful algorithm. Admittedly not all the known algorithms have been compared to the SGLDM to prove its absolute superiority but nonetheless these studies indicate it is relatively powerful based on all known comparative data.

The final supportive data for Assumption 1 comes from the many successful uses of the SGLDM to solve applications problems. References [4, 5, 6, 15, 16] are just a few such examples. This record of performance on real world data would also seemingly indicate the power of the SGLDM.

Unfortunately there is also some growing evidence indicating that second-order probabilities may not be a good model for the primitive mechanisms of human perception. Obviously, such evidence necessarily casts doubt on the overall robustness of the SGLDM. This evidence is the growing number of counterexamples to the Julesz conjecture which have appeared in the literature since 1977 [17, 18, 19, 20]. These counterexamples, some of them quite striking, are visually distinct texture pairs which have identical second-order probabilities.

Because of the existence of these counterexamples seemingly directly challenges the validity of Assumption 1, the basic assumption upon which all of our work has been based, a study of these counterexamples was conducted during the last funding year. The precise results of this study will be reported in the next subsection of this progress report. It suffices here to say that it was found that the SGLDM could discriminate each of these counterexamples using the principle of what we call global/local analysis. This was part of subtasks 1 and 2 of the renewal proposal.

An important point to bring out at this time is that there is no known example of a visually distinct texture pair which cannot be discriminated by the SGLDM.

This leads us to the second assumption.

Assumption 2: Information concerning the visual qualities of patterns can be reliably obtained from the spatial gray level dependence matrices only when these matrices are computed for several different values of θ and several different consecutive values of d for each θ considered.

It is important to note that the above assumption makes no statement about the number of d and θ values which need be considered since obviously these numbers are problem dependent.

In presenting arguments to support this assumption, let us begin by examining the visually distinct texture pair shown in Figure 3. This Markov texture pair was generated using a precedure described in [2]. The transition matrices used to create these textures are given in Table 1.

The interesting point about these textures is that only two angular orientations, namely, $\theta = 0°$ or $\theta = 180°$, will allow the SGLDM to discriminate these textures. Even when $\theta = 0°$ or $\theta = 180°$ an intersample spacing distance of $d = 2$ will yield identical expected values for the spatial gray level dependence matrices computed from these textures. That is $E\{S^{(1)}(\delta)\} = E\{S^{(2)}(\delta)\}$ $\delta = (2,0°)$ or $\delta = (2,180°)$ where $S^{(1)}(.)$ represents the spatial gray level dependence matrices extracted from one texture and $S^{(2)}(.)$ represents the spatial gray level dependence matrices extracted from the other texture in the pair. What this example and many other easily generated examples indicate is that several different θ values may have to be considered to find those directions which will allow discrimination and that given a particular θ no single d value will yield a spatial gray level dependence matrix which has much information concerning the visual qualities of the pattern.

Other supporting evidence for this assumption comes from studies conducted on real world data. Weszka, Dyer and Rosenfeld [12] reported

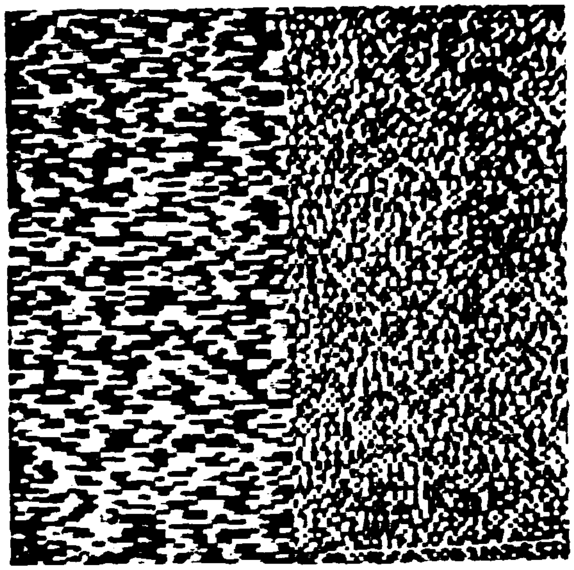


Figure 3. Two textures which can only be discriminated in the $\theta = 0^{\circ}$ or $\theta = 180^{\circ}$ directions. Further even in these two directions a d value of 2 will yield identical expected values for the spatial gray level dependence matrices computed from these textures.

$$(A)\quad \begin{bmatrix} .6 & 0 & 0 & .4 \\ .4 & .6 & 0 & 0 \\ 0 & .4 & .6 & 0 \\ 0 & 0 & .4 & .6 \end{bmatrix}$$

$$(B)\quad \begin{bmatrix} 0 & .4 & .6 & 0 \\ 0 & 0 & .4 & .6 \\ .6 & 0 & 0 & .4 \\ .4 & .6 & 0 & 0 \end{bmatrix}$$

$$(C)\quad [\ .25 \quad .25 \quad .25 \quad .25]$$

Table 1. The Markov transition matrices used to generate the two textures in Figure 3. (a) The transition matrix for creating the texture on the left. (b) The transition matrix for the texture on the right. (c) The initial distribution used to pick the first element of every row. The synthesis procedure used to generate these textures is described in [2].

improvements in classification accuracies when multiple d values were used. The data base for this study were aerial photographs in one instance and Landsat data in another. In [6] the authors reported improvements in classification accuracies in classifying lung infiltrates on chest radiographs when multiple d values were used. Also Kruger et al [15] used two d values in a study involving the automatic classification of Coal Workers' Pneumoconiosis. Presumably the only reason why two d values would be used would be because classification accuracies were improved. In all the above studies only four values of θ were considered, namely 0°, 45°, 90° and 135°.

Still more evidence supporting Assumption 2 comes from the authors' study [1] which showed that if enough d and θ values were considered then the periodicity of a texture could be determined as well as the size, shape and placement rule of a special unit pattern called the period parallelogram unit pattern. In particular, it was shown that plots of the inertia feature

$$I(\delta) = \sum_{i}\sum_{j} (i-j)^2 \, s(i,j,\delta) \tag{2}$$

as a function of d for a given θ allow one to detect periodicity in the θ direction. Considering such plots for several values of θ allow one to determine the size, shape and placement rules of the period parallelogram unit pattern, i.e., the vectors $\vec{a}$ and $\vec{b}$ shown in Figure 4.

With the above in mind there is one other area of perceptual research that needs to be mentioned. It has been known for sometime that the human eye is always in motion. Even under conditions where steady fixation is attempted, small, involuntary movements of the eye are always present. Knowledge of these movements has generated experimental attempts to understand their role in the visual process. The questions which these investigations have attempted to answer are: (a) What is the nature and extent of the involuntary eye movements? (b) What effect does these movements have on

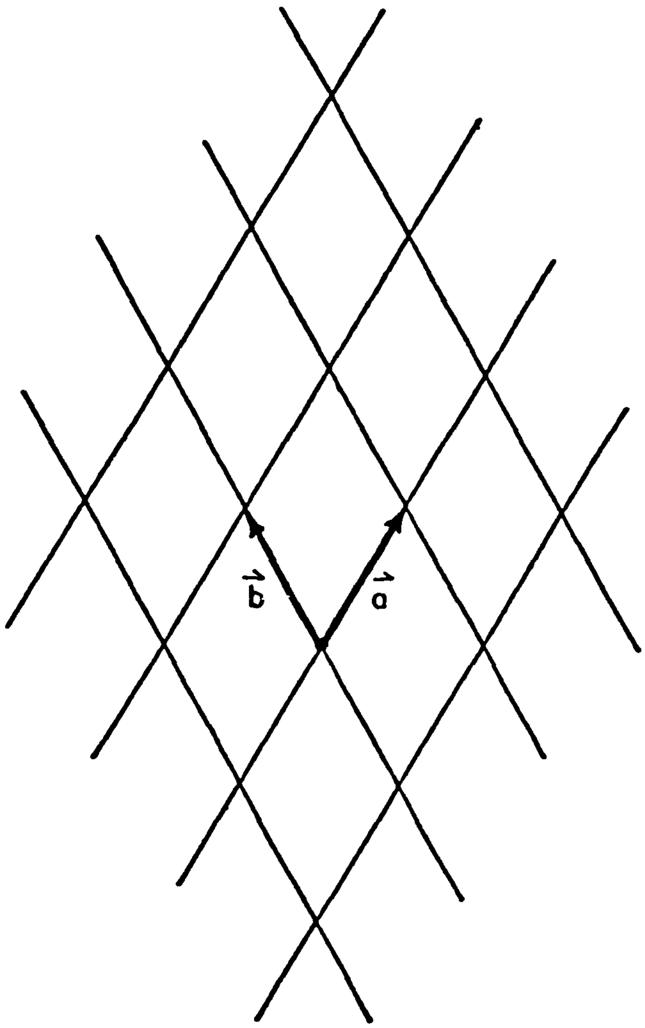


Figure 4. The vectors $\bar{a}$ and $\bar{b}$ completely define the size and shape of the period parallelogram unit pattern. Further these two vectors define the placement rules for this unit pattern.

the visual process?

Ratliff and Riggs [21] provide an answer to the first question. Their results indicate that the involuntary movements consist of: a slow drift of the eye; a rapid, jerking movement (saccadic); and a small, rapid tremor superimposed on the drift.

The experimental methods which have been used to attack the second problem have been to stabilize the retina image. The point of interest has been to determine the effects on the visual process of reducing or stopping the involuntary movements of the eye. Heckenmueller [22] provides an excellent review of these experiments.

While this area of investigation has been the subject of some controversy, one fact seems clear; when the image of a visual target is stabilized on the retina, the image disappears and gives way to a homogeneous field. This fact has lead investigators to believe that the effect of involuntary eye movements is primarily one of overcoming the loss of vision resulting from constant stimulation of the retina. That is, the effect of the movements is to provide changing sensory stimulation of the spatial variety.

The reason for mentioning these studies is that they indicate the eye is responding not to absolute light intensity values but rather differences in intensity over some spatial movement. That is, the eye seems to be responding to changes from gray level i to gray level j given a spatial displacement δ. Of course, this is precisely the type of variation that is being measured by the spatial gray level dependence matrices. The rapid eye movements can be interpreted as measuring these spatial variations in gray level over numerous directions θ and magnitudes of displacement d. This interpretation, albeit it a loose one, seems to imply that the underlying assumptions stated above may, indeed, be based on the actual human processes of vision.

Now that the basic assumptions have been stated, evidence supporting these assumptions presented and documentation given that these concepts may be used in the human visual process, it seems appropriate to define the structure of the SSA. Actually a precise definition of the structure of the SSA at this juncture in time is not possible. The SSA is still under development and additional information is needed before any degree of precision can be added to the hierarchy the analyzer will take. However, what is known is that the SSA has at least two levels of features, Level 0 and Level 1. Level 0 features are the primitive features which drive the whole system. An example of a Level 0 feature is the inertia measure computed for a particular δ. From the above it should be clear that a Level 0 feature, by itself, contains little information about the visual quality of a pattern. However, a Level 0 can be used to do classical texture discrimination. That is, given a sample region A these features can be used to determine whether the pattern of region A belongs to one of, say, n given classes of patterns. Examples of this classical type discrimination are given in references [4, 5, 6, 12, 15, 16]. This type of analysis usually employes statistical feature selection and pattern recognition procedures. Further, this type discrimination is such that given the automatically selected features and decision boundaries it is usually impossible to correlate these features with any visual qualities of the patterns involved.

The Level 1 features of the SSA are computed not from the pattern but rather from the Level 0 features. A Level 1 feature, for example may be computed from values of the inertia measure obtained for several d and

θ values. Examples of four Level 1 features would be the coordinates of the vectors $\vec{a}$ and $\vec{b}$ which define the size, shape and placement rule of the period parallelogram unit pattern of a periodic texture. It is the Level 1 features which give information about the visual qualities of patterns.

Level 1 features should provide us advanced capabilities. In particular, they would seem required in determining texture similarity. They can be used to guide image segmentation, i.e., they can be used to determine not only that a split is needed but also what the region size of each new region should be, for example, as in segmenting an image composed of two or more textures all of which have the same periodicity. They would seemingly provide a convenient mechanism by which semantic information about textures could be incorporated into the analysis process.

At this point in time of our development effort, only one Level 0 feature has been defined. This is the inertia feature. It is a bonafied Level 0 feature because in [1] it was shown that this feature could be used to measure visual qualities of patterns. It is also known that other Level 0 features are needed. Figure 5 shows two visually distinct texture patterns which have identical values for the inertia feature for any given δ. Consequently, this texture pair cannot currently be discriminated by the SSA, though the spatial gray level dependence matrices contain information which will allow their discrimination.

The goal now must be to define more Level 0 features, features which will given the system greater capability. Since the capability of the SSA clearly rests on having a number of quality Level 0 features, the development of this feature set must be done very carefully. Our goal is to remove as much as possible the role of heuristics in defining any new Level

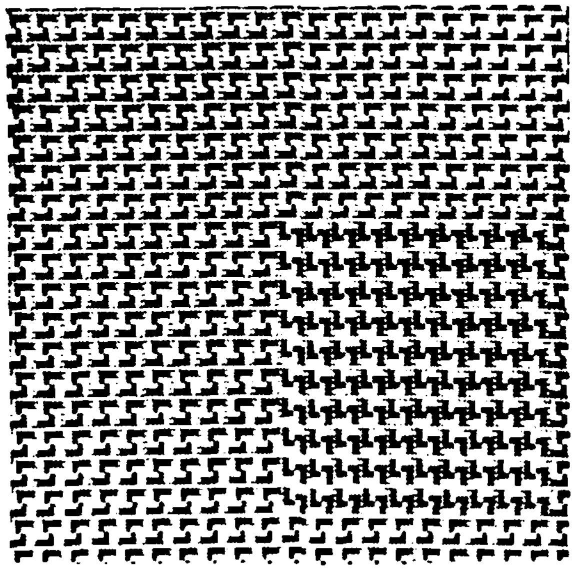


Figure 5. Two visually distinct textures which cannot be discriminated by the inertia measure. It should be added that information is available in the spatial gray level dependence matrices which will allow the discrimination of these two textures.

0 features. The idea is to create a method for generating new Level 0 features based on experimental parametric methods.

B. *On Discriminating Textures Which Have Identical First- and Second-Order Probabilities*

Assumption 1 would seem based in part on the validity of the Julesz conjecture. Since recently there has been a number of rather striking counterexamples to this conjecture, it seemed advisable to examine these counterexamples in some detail. The idea is to check the robustness of the SGLDM algorithm. If this algorithm cannot discriminate the known counterexamples to the Julesz conjecture then one must wonder whether the continued development of any new approaches to texture analysis based on the SGLDM would be appropriate.

In what follows we will provide a brief background on the investigations related to the Julesz conjecture. In particular we will concentrate on the synthesis procedures which have been devised to generate texture pairs which have identical first-and second-order probabilities. Finally we will present our analysis methods used to discriminate these textures.

To begin it seems appropriate to review the origins of the Julesz conjecture, itself. This conjecture represents the results of a 1962 study [7] to determine the smallest value of n such that if two textures A and B have identical n^{th}-order probabilities but different (n+1) - order probabilities, no human discrimination of these textures would be possible. To perform this study required a synthesis procedure for creating textures with controllable n^{th}-order probabilities. The procedure used was developed by Rosenblatt and Slepian [23]. Unfortunately the Rosenblatt and Slepian synthesis procedure could generate only a very limited class of textures.

Because the results of the 1962 study were based on such a limited class of experimental data, the validity of the Julesz conjecture has always been somewhat suspect. Consequently, a number of investigators have attempted to disprove the conjecture by generating counterexamples of its validity. Two early attempts were done by Pollack [10] and Purks and Richards [11]. Unfortunately these investigators did not consider the most general form of the Julesz statement of his conjecture and hence their examples texture were not true counterexamples [8].

Other studies aimed at determining the validity of the Julesz conjecture were reported by Pratt, Faugeras and Gagalowicz [82] and Julesz, Gilbert and Shepp [9]. Pratt et al expanded on Julesz's original work [7] by using the Rosenblatt and Slepian synthesis method. They also developed a new method for synthesizing texture pair which have identical second-order probabilities. However, none of the experimentation done using these methods yielded a counterexample.

Julesz, Gilbert and Shepp developed four new synthesis methods for use in their study. The proofs that these methods generate texture pairs which have identical first-and second-order probabilities is given in a report by Gilbert and Shepp [24]. Each of the methods used generate a texture by using a single "micropattern" whose placement is controlled by two operations, i.e. a translation and perhaps a rotation. The translations used were such that the individual micropatterns did not overlap and further the micropatterns were at least some minimum distance apart. If random rotation were used, the angles were chosen independently with angles distributed with constant density in $[0, 2\pi]$.

Extensive experimentation using the new synthesis methods did not yield any successful counterexamples. The experimental procedure employed required that the texture pairs be discriminable within 200 ms. This procedure was used since the investigators were interested in spontaneous discrimination, i.e., that which can be done without scrutiny.

One of the synthesis methods used is called the method of 180° rotations. This method uses micropatterns α and its dual β derived by rotating α by 180°. The textures A and B are derived from α and β, respectively, by using the same translations to produce a pair of textures with identical second-order probabilities. Random rotations are not used. A texture pair generated using this method is shown in Figure 6. Note that while the textures cannot be discriminated spontaneously they can be discriminated with scrutiny.

Another synthesis method used in [9] is called the method of mirror transform. Using this method the micropattern used to generate texture B is a mirror image of the micropattern used to generate the texture A. The same translation is used to place the micropatterns in both textures. The patterns must be randomly rotated to give textures with identical second-order probabilities. Figure 7 shows a texture pair generated using this synthesis method. Obviously these textures cannot be discriminated spontaneously and even a careful examination of the image does not yield a good discrimination.

Another synthesis method used in [9] is called the 4-disk method. The way the two micropatterns are constructed in this method is shown in Figure 7A. A triangle KLM is drawn. In the middle of one of its sides, say, LM, a point O is placed. Connecting O with K form a line segment OK. At the point O a line perpendicular to OK is drawn. Finally, two points P and Q are selected on this line such that PO = QO. One micropattern consists then of four nonoverlapping disks with their centers at the points K, L, M and P while the other micropattern consists of

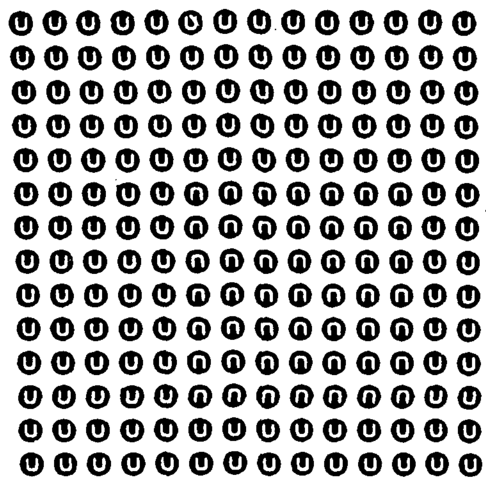


Figure 6. Two textures created using the method of 180° rotation. These two textures have identical secondporder probabilities and are not spontaneously discriminable by human observers. They can however be discriminated upon a more careful examination.

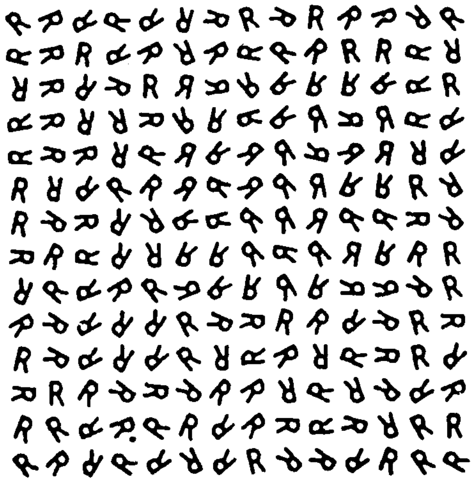


Figure 7. Two textures generated using the method of mirror image. These two textures have identical second-order probabilities and are not spontaneously discriminable by the human observer. Discrimination of these textures is difficult even upon a careful examination of the image.

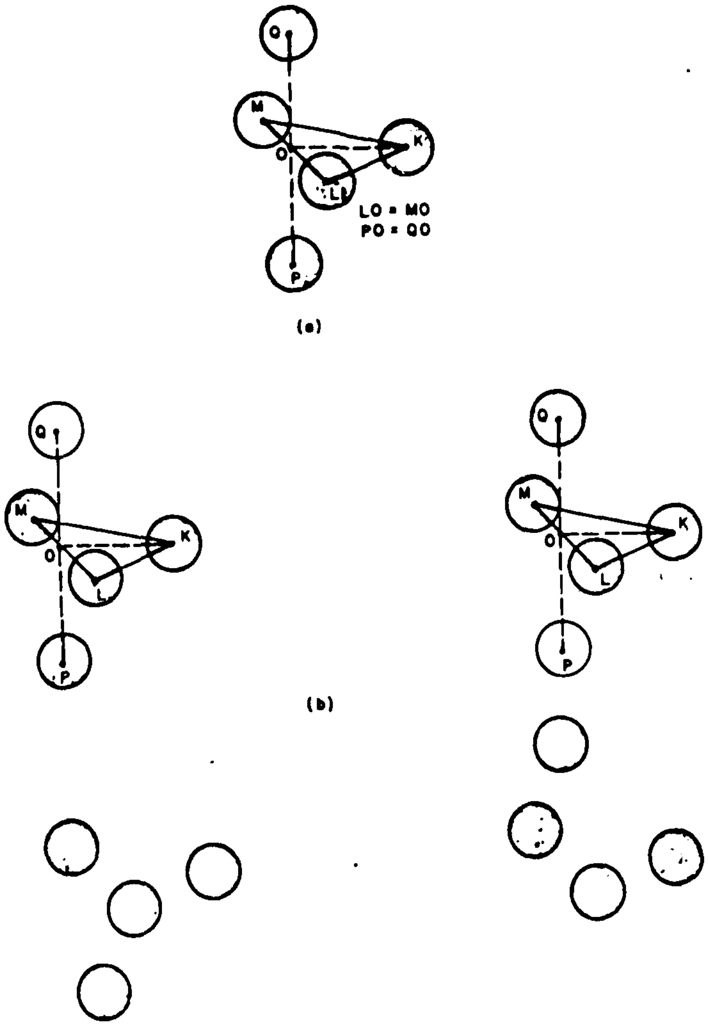


Figure 7A. The 4-disk method for generating textures which have identical second-order probabilities. (a) An example of the geometry used in the 4-disk method. (b) Shows how the two micropatterns are generated from this geometry. (c) Shows the resulting micropatterns.

the same disks centered at K, L, M and Q. These micropatterns are then placed using the same set of translations. One micropattern is used to generate texture A and the other is used to generate texture B. Random rotations of the micropatterns are required to generate textures with identical second-order probabilities.

The advantage of this synthesis method over the other two is that one can find micropatterns which appear very different. An example texture pair generated using this procedure will be given in Figure 8. The texture pair in Figure 8 was one of those examined by Julesz et al in [9]. While these textures are not spontaneously discriminable, a careful examination will yield a good discrimination.

The fourth method described by Julesz, et al, in [9] is the one that yielded the weak counterexample. However, this synthesis procedure is very limited. Consequently we will not discuss it. However, the weak counterexamples are shown in Figure 9.

The next research of interest was reported by Caelli and Julesz [17] and Caelli, Julesz and Gilbert [18]. These papers are of interest because

a) they presented the first good counterexamples to the Julesz conjecture;

b) they attempted to explain the human discrimination of textural patterns which have identical second-order probabilities;

c) and they presented some interesting techniques for both isolating the counterexamples and in studying the problem to arrive at an explanation of the human discrimination of these counterexamples.

Concerning the last point Caelli, Julesz and Gilbert used Monte Carlo methods to find counterexamples. For example the 4-disk method has a number of variable parameters such as the positions of the points K,L, and M. These positions were randomly varied and the resulting texture pairs were examined to determine whether they were visually distinct.

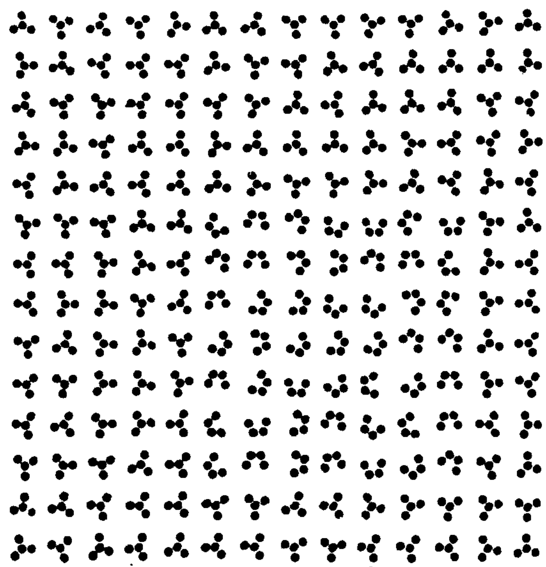


Figure 8. A texture pair generated using the 4-dish method. While these textures are not spontaneously discriminable, i.e., discrimination is less than 200 ms, they can be easily discriminated under a more careful examination.

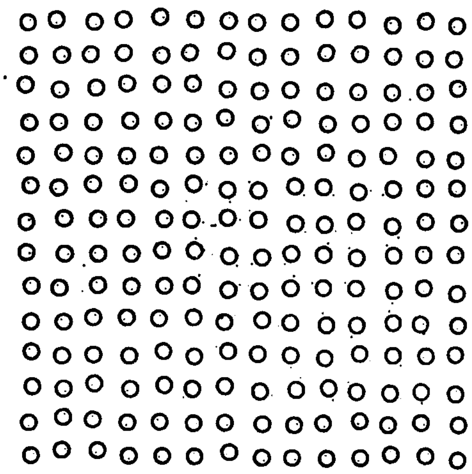


Figure 9. The "weak" counterexample of the Julesz conjecture presented by Julesz, Gilbert and Shepp in [9].

Also in formulating their explanation for the discrimination of texture pairs which have identical second-order probabilities, the investigators followed another interesting series of steps. These are outlined as follows:

1. Using a particular synthesis procedure randomly generate texture pairs which have identical second-order probabilities. The purpose of this step is to isolate a set of texture pairs which are spontaneously discriminatable to the human observer.

2. Using this set of visually distinct textures attempt to isolate the visual quality or qualities that seemingly explain the discrimination of these textures.

3. If step 2 is possible, synthesis texture patterns with varying degrees of this visual quality and show that as this visual quality becomes more pronounced the discrimination becomes more obvious.

Also in [17] and [18] two new texture synthesis methods were introduced. First of these is the 5- and 6-disk method. In this discussion only the 6-disk formulation will be presented since the 5-disk synthesis technique is a degenerate case of the 6-disk method. In this procedure the centers of the 6-disks of one micropattern are defined by six complex numbers $[0,1,a,b,c_1,c_2]$ where it is required that the real components of a and b sum to one, $\mathrm{Re}\{a+b\} = 1$, and that imaginary components of a, b, c_1 and c_2 satisfy $\mathrm{Im}\{a + b - 2c_i\} = 0$, $i = 1,2$. The centers of the 6-disks comprising the other micropattern are then given by the complex numbers $[0,1, 1-a, 1-b, c_1^*, c_2^*]$ where c_i represents the complex conjugate of c_i. Each micropattern is used to generate one texture in the texture pair. To generate a texture the micropattern is regularly placed and randomly rotated. The resulting texture pair has textures with identical second-order probabilities.

The other new synthesis method developed is called the "most general" method. This procedure will generate micropatterns containing any number of disks or even elements which are not disk but arbitrary shapes and contours. This procedure is a generalization of the 4-disk method described above. The most general procedure uses three "sets" which correspond to the disks centered at the points K, P and the pair (L,M) of the 4-disk method. Each set represents either a single object such as a disk or a collection of objects such as several disks. The only requirement is that object or objects comprising the set satisfy certain symmetry requirements. For example the set corresponding to the disk centered at P and Q must be reflectively symmetric about the line passing through P and Q, i.e, PQ. Let S be the point of intersection between PQ and a line perpendicular to PQ passing through K. The set corresponding to the disk at K must be reflectively symmetric about the line KS. Finally the set corresponding to the combination of the two disks centered at points L and M must be rotationally symmetric about the point S for a rotational angles of 180°.

In this procedure one micropattern is composed of the sets centered at K, L, M and P and the other micropattern of the sets centered at K, L, M and Q. The textures are generated by regularly placing and randomly rotating the patterns.

Using these two new procedures together with the 4-disk method the investigators were able to locate a number of good counterexamples to the Julesz conjecture. Further, they used these counterexamples to formulate a new conjecture concerning spontaneous human discrimination of texture. The new conjecture states that the human texture discrimination system is based on two types of perceptual analyzers (or detectors): Class A and Class B. The Class A analyzers are determined by differences in second-order probabilities or as Julesz refers to them as dipole statistics.

The Class B analyzers refer to those features the visual system may extract independent of second-order statistics, in particular, when the second-order statistics are equal.

In [17] and [18] a total of three Class B detectors were found. These were the quasi-collinearity detector, the corner detector and the closure detector. To defend the existence of the Class B detectors, Caelli and Julesz [17] state, "texture discrimination cannot be a strictly statistical problem but obviously the inherent structures of the visual system must be taken into account."

Caelli, Julesz and Gilbert [18] make one point very clear. They state,

> "... the reader should be reminded that out of the thousands of iso-dipole texture pairs that were generated at random by any of the known methods only a tiny fraction yielded discrimination, and out of these rare events did we select our demonstrations. Therefore, it is regrettable that the reader was not shown an adequate number of typical examples of the vast majority of iso-dipole texture pairs that were found not to be effortlessly discriminable."

All the synthesis procedures mentioned thus far have been designed to give texture pairs with identical second-order probabilities. They cannot be used to create texture pair which have identical higher-order probabilities, i.e., third-order, fourth-order, etc. The next snythesis procedure described gives texture pairs with identical third-order probabilities and hence identical second-and first-order probabilities. This procedure was described in a 1978 by Julesz, Gilbert and Victor [19]. The motivation for creating this new synthesis method was to continue the search for more Class B detectors.

This synthesis method generates what are termed even and odd textures. Consequently we will refer to this method as the Even-Odd Method. To define what constitutes an "even" or an "odd" texture let $g(i,j) = 1$ if the

i,j^{th} pixel is black and $g(i,j) = 0$ if this pixel is white. An even texture is one where

$$g(i,j) + g(i+1,j) + g(i,j+1) + g(i+1,j+1) \qquad (3)$$

is an even number for all i,j. An odd texture is one where Eq. 3 gives an odd number for all i,j.

Thus texture pair shown in Figure 10 is an example of these even and odd textures. The texture on the left is an even texture and the texture on the right is an odd texture.

Note that while these textures have identically the same first-, second-and third-order probabilities they can easily be visually discriminated. Julesz et al [19] attribute the discrimination of the even and odd textures to the existence of a Class B detector, called the granularity detector.

To generate either the even or odd texture one selects an arbitrary row and column of the image to paint first. Each element of this row and column is independently assigned a gray level according to the probability law $P(0) = \frac{1}{2}$ and $P(1) = \frac{1}{2}$. For convenience assume that row 0 and column 0 are the ones that were picked. Note that the element 0,0 only needs to be painted once.

After row 0 and column 0 have been painted the rest of the image is assigned gray levels. An even texture is created by using the equation.

$$g(i,j) = [g(i,0) + g(0,j) + g(0,0)] \bmod 2.$$

An odd texture is created by using the equation

$$g(i,j) = [g(i,0) + g(0,j) + g(0,0) + ij] \bmod 2.$$

The last study of interest was the one conducted by Gagalowicz [20]. The purpose of the study, like the others, was to find and analyze

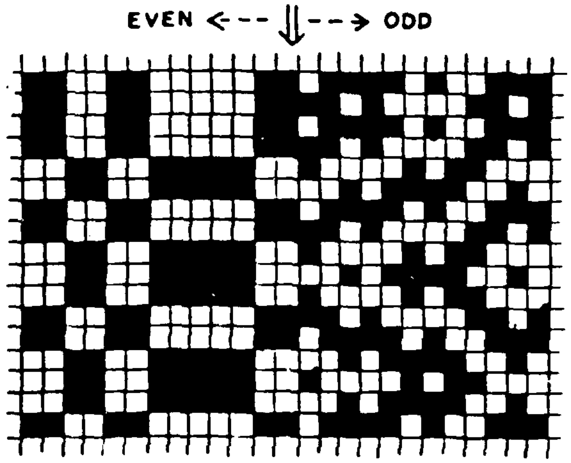


Figure 10. A texture pair composed of an even texture (on the left) and an odd texture (on the right). The two textures comprising this texture pair have identical first-, second-, and third-order probabilities.

counterexamples to the Julesz conjecture. What makes Gagalowicz's study of interest is (a) he developed an interesting synthesis procedure for generating counterexamples and (b) his examination of the counterexamples he generated lead him to a different set of conclusions about the factors involved in the discrimination of these textures.

As to the Gagalowicz synthesis method, this procedure has three input parameters. These are the number of gray levels, L, to appear in the texture, the number of pixels, K, to appear in each elementary building block or tile of the texture, and finally the geometry of this building block or tile. For example, one possibility is K horizontally adjacent picture points with L possible gray levels. Here K is the pattern size and the horizontal adjacency gives the shape of the pattern. For this example there are a total of K^L possible patterns or tiles. The idea is to tile in the plane using these patterns so that there are no holes or overlaps. The choice of which of the K^L patterns to place at any given location is done randomly by independently selecting the pattern according to a probability law governing the frequency of occurrance of each possible pattern. The trick to the procedure is the mechanism used for computing the probability law so that the second-order probabilities of the resulting pattern are what they are desired to be.

To see how the probability law is computed we will introduce the following conventions. The subset D of I^2, the two-dimensional integer lattice, represents the pattern shape to be used in the generation process. It will be assumed that domain of X(n,m) is D which is finite, namely containing only K points.

The second-order probabilities for X(n,m) are the probabilities $P[(X_i = L_i), (X_j = L_j)]$ for X to be equal to L_i in position i, and to be equal to L_j in position j, for all positions $i,j \varepsilon D$. Note we have dropped the double arguments for X(n,m) for notational convenience where X_i means X evaluated at one of the K points at which it is defined.

It is possible to synthesize the process X in the finite domain D of K points, if we know its K^{th}-order probabilities,

$$P[(X_1 = L_1), (X_2 = L_2), \ldots, (X_K = L_K)]$$
$$= P(L_1, L_2, \ldots, L_K).$$

Here $P(L_1,L_2,\ldots,L_K)$ are the probabilities for X to be equal to L_1 in position 1, to be equal to L_2 in position 2,..., to be equal to L_K in position K.

The objective is to construct $P(L_1L_2,\ldots,L_K)$ from a priori given second-order probabilities $P[(X_i = L_i), (X_j = L_j)]$. For convenience we write $P[(X_i = L_i), (X_j = L_j)]$ as $P_{ij}(L_i,L_j)$.

To see how this can be done one need only remember that the K^{th}-order probabilities are constrained by the second-order probabilities. These constraints take the form of partial summations. In particular

$$P_{i,j}(L_i,L_j) = \sum_{L_1=0}^{L-1} \sum_{L_2=0}^{L-1} \cdots \sum_{L_{i-1}=0}^{L-1} \sum_{L_{i+1}=0}^{L-1} \cdots \sum_{L_{j+1}=0}^{L-1} \sum_{L_K=0}^{L-1}$$
$$P(L_1,L_2,\ldots,L_{i-1}, L_i, L_{i+1},\ldots, L_{j-1}, L_j, L_{j+1}\ldots L_K) \qquad (3)$$

for i,j, $i,j \varepsilon \{1,2,\ldots,K\}$, and $L_m \varepsilon \{0,1,\ldots,L-1\}$, $m = 1,2,\ldots,K$. There are exactly $L^2C_2^K$ such equation constraints where C_2^K represents the number of possible combinations of K things taken 2 at a time.

Further constraints are placed on the K^{th}-order probabilities, since the first-order probabilities $P(X_i = L_i) = P_i(L_i)$. These first-order probabilities must also be obtainable from the K^{th}-order probabilities through partial summations.

These constraints are

$$P_i(L_i) = \sum_{L_1=0}^{L-1} \sum_{L_2=0}^{L-1} \cdots \sum_{L_{i-1}=0}^{L-1} \sum_{L_{i+1}=0}^{L-1} \cdots \sum_{L_K=0} P(L_1, L_2, \ldots, L_{i-1}, L_{i+1}, \ldots, L_K) \tag{4}$$

There are exactly LC_1^K such equation constraints where C_1^K represents the number of combinations of K things taken 1 at a time. It should be noted that the first-order probabilities $P_i(L_i)$ are directly obtainable from the a priori given second-order probabilities $P_{i,j}(L_i, L_j)$, through another partial summation process.

The final set of constraints on the K^{th}-order probabilities are

$$\sum_{L_1=0}^{L-1} \sum_{L_2=0}^{L-1} \sum_{L_K=0}^{L-1} P(L_1, L_2, \ldots, L_K) = 1 \tag{5}$$

and

$$P(L_1, L_2, \ldots, L_K) \geq 0 \tag{6}$$

for $L_1, L_2, \ldots, L_K \in \{0, 1, \ldots, L-1\}$. Note that there are K^L constraints of the form given by the inequalities of Equation 4.

Finally, it should be noted that these constraints don't usually uniquely specify the K^{th}-order probabilities. Indeed usually there are infinitely many solutions which satisfy these constraints assuming, of course, K and L are sufficienlty large. It should also be noted that the number of constraints grow rapidly in relation to increases of K and L.

In recognition of these problems Gagalowicz developed a scheme for reducing the total number of constraints by removing all the redundant ones. In particular he was able to reduce the number of constraints of the type given in Equation 3 from $L^2C_2^K$ to $(L-1)^2C_2^K$. Further he was able to reduce the constraints of the type given in Equation 4 from LC_1^K. Secondly he cast the problem of determining the K^{th}-order probabilities in terms of the a priori given second-order probabilities as a linear programming problem. What gives uniqueness to the solution process in the appropriate albeit arbitrary selection of a cost functional. (For more information on linear programming in general and the simplex method in particular see [25] and [26].)

Using this synthesis method Gagalowicz generated two counterexamples to the Julesz conjecture. One of these is shown in Figure 11. In this example K=7 and L=2. The pattern geometry was 7 horizontally adjacent points. The pattern on the left was generated with the pattern probability assignments P(1,1,1,1,1,1,1) = 1/8, P(1,1,2,1,2,2,2) = 1/8, P(1,2,1,2,2,1,2) = 1/8, P(1,2,2,2,1,2,1) = 1/8, P(2,1,1,2,1,2,2) = 1/8, P(2,1,2,2,2,1,1) = 1/8, P(2,2,1,1,2,2,1) = 1/8 and P(2,2,2,1,1,1,2) = 1/8 where 1 is black and 2 is white. The pattern on the right was generated with each of the 128 patterns having a probability of occurrence of 1/128. The cost functional used to generate the probability of occurrences of the pattern on the left was one which maximized the magnitude of P(1,1,1,1,1,1,1). Both the textures in Figure 11 have identical second-order probabilities with each equalling 1/4.

Concerning his experiments using his synthesis procedure, Gagalowicz points out that all attempts made to generate homogeneous looking textures which have identical second-order probabilities resulted in texture pairs which were not discriminable to the human observer. The only counterexamples which he was able to generate were those in which at least one of the two textures had inhomogeneities, i.e. such as the black lines appearing in the texture on the left in Figure 11. The more inhomogenieties that occur the better the visual discrimination. Further, he states that the homogeneous background areas of the textures usually look the same for both

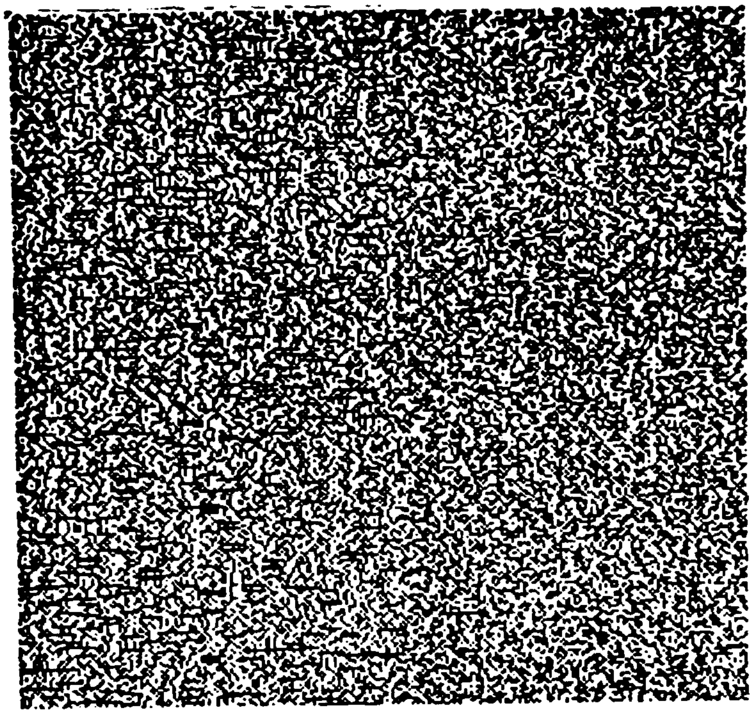


Figure 11. Two visually distinct texture pairs which have identical first- and second-order probabilities. These textures were generated by Gagalowicz in [20].

textures which explains the difficulty in determining the exact boundary between the textures. (Again refer to Figure 11.)

The above observations lead Gagalowicz to a hypothesis about the visual discrimination process. He claims that in those instances when textures which have identical second-order probabilities are not discriminable, the second-order probabilities computed locally correspond well to second-order probabilities computed globally. But, he continues, when inhomogeneities are present the local second-order probabilities disagree with the global second-order statistics. It is this difference that is perceived by the eye and used for discrimination.

The last synthesis procedure to be discussed is a very limited method that was developed a few years ago by the authors. The reason for presenting this procedure is that it generates a counterexample to the Julesz conjecture which will prove useful in our discussion of our analysis methods.

To describe how this synthesis method works consider the patterns shown in Figure 12. The two patterns in part (a) of the figure are used to generate one texture while the four patterns in part (b) are used to generate the other texture in this counterexample. The generation procedure is a simple one and the same procedure is used to generate each row. The first element of each row is selected according to the probability law $P(i) = 1/n$ where i represents the i^{th} pattern and n represent the total number of patterns being used to generate that texture, i.e. $n = 2$ or 4. The rest of the row is completed by merely repeating the first element of the row. Figure 13 shows a texture pair generated by this procedure. These two visually distinct textures have identical second-order probabilities.

The background information presented above leaves little doubt about

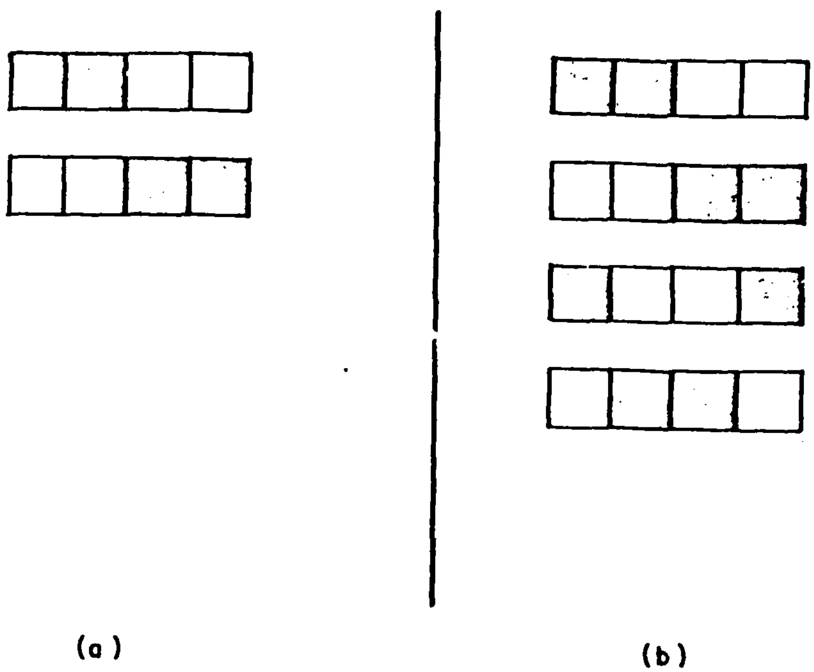


Figure 12. The patterns used to generate a texture pair which have identical second-order-probabilities. The patterns in (a) are used to generate one texture; the patterns in (b) are used to generate the other texture.

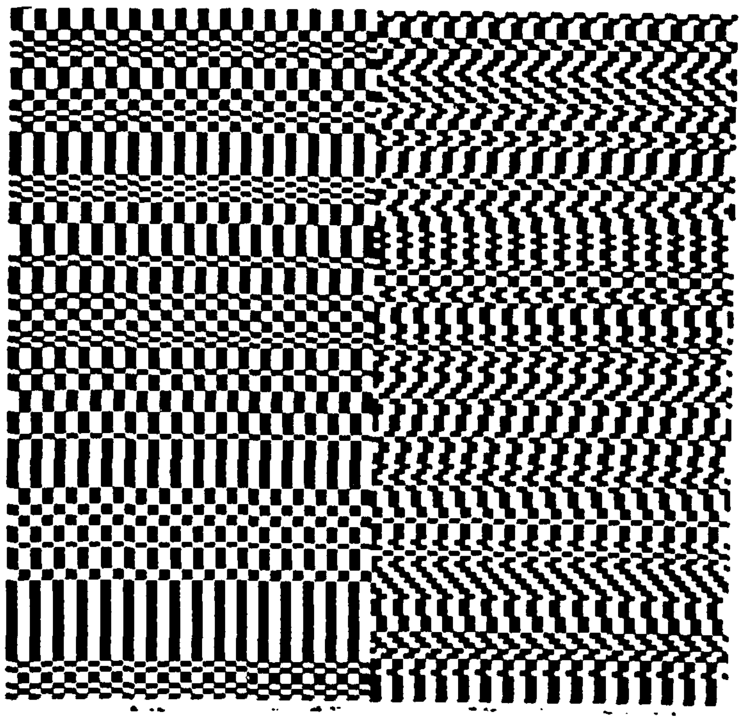


Figure 13. A texture pair generated using authors' generation procedure.

a number of points. First, all investigators involved in the search for counterexamples to the Julesz conjecture have reported that the vast majority of texture pairs which have identical second-order probabilities are not discriminable by the human visual system. These observations are supported by the number of successful uses on natural occurring textures of the SGLDM algorithm and other texture algorithms which measure all or part of the second-order probabilities. Next it is quite clear that there are some very strikingly visually different textures which have identical second-order probabilities. Consequently, the known counterexamples to the Julesz conjecture cannot be ignored even in light of the difficulty of producing these examples. Finally, there is substantial disagreement as to how the human visual system goes about the discrimination of the known counterexamples.

In what follows we will show how these counterexamples, or at least a representative subset of the counterexamples, can be discriminated by the SGLDM. The method of analysis can be termed a global/local procedure where first a global analysis is done and then a local analysis is performed using the same basic methodologies as were used globally. This approach was impart suggested by Gagalowicz [20]. This method of procedure, i.e., applying the same techniques both globally and locally has in the authors' minds more of an esthetic appeal than the concept of Class A and Class B detectors. Further, and this is important, the global/local analysis methods used are exactly the same type as those required to analyze a simple periodic texture. The procedure used on periodic textures was described in [1] and required a global analysis to impart find the period parallellogram unit pattern. Then a local analysis is required on the various unit patterns. As such the methods used to discriminate the counterexamples do not represent special or unusual techniques arrived at just to discriminate these textures but rather represent a natural procedure required to discriminate any texture.

For purposes of this discussion we will lump all the known counterexamples into two groups. The first group is called the pure group. A texture pair is in the pure group if there are no common patterns appearing in the two textures comprising the pair. Texture pairs generated by the 4-disk; 5- and 6-disk; most general; and even/odd methods are in the pure group.

On the other hand a texture pair is in the impure grouping if the two textures do share common elements. Texture pairs generated by the Gagalowicz procedure and the authors' synthesis method will be in the impure group. It should be noted that it might theoretically be possible for the Gagalowicz method to yield texture pair in the pure group but the two counterexamples he has presented thus far both are in the impure group.

Differentiating the two groups is done because one would expect and want a texture pair in the pure group to be perfectly discriminated. That is, one would expect the classification accuracy to be 100% since the two textures comprising the pair do not share any common patterns. On the other hand perfect discrimination of impure texture pair is impossible. One may get classification accuracies arbitrarily close to 100% but perfect classification is impossible.

We will begin by considering the pure group of counterexamples. In particular we will begin by considering a counterexample generated by using the 4-disk method. This counterexample is shown in Figure 14.

It is important to make a few observations. First the 4-disk, 5-

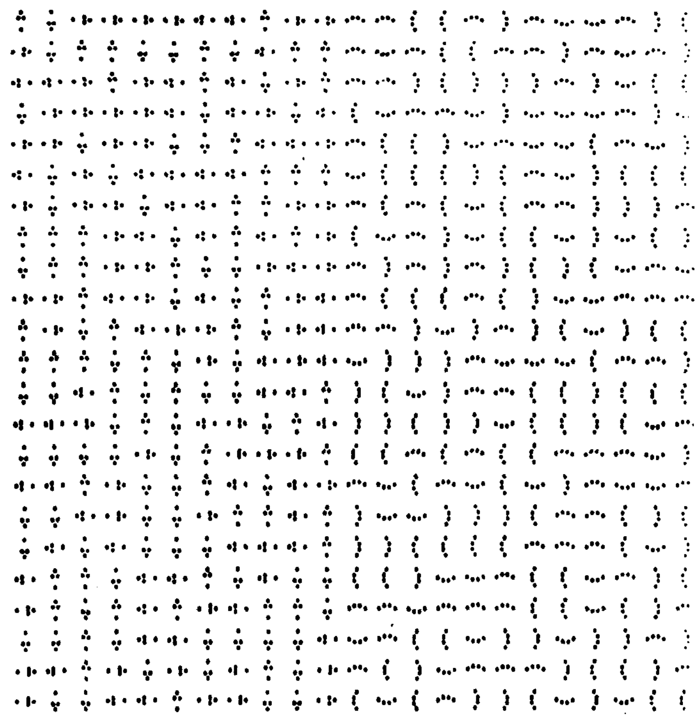


Figure 14. A visually distinct texture pair generated using the 4-disk method. These two textures have identically the same first- and second-order probabilities.

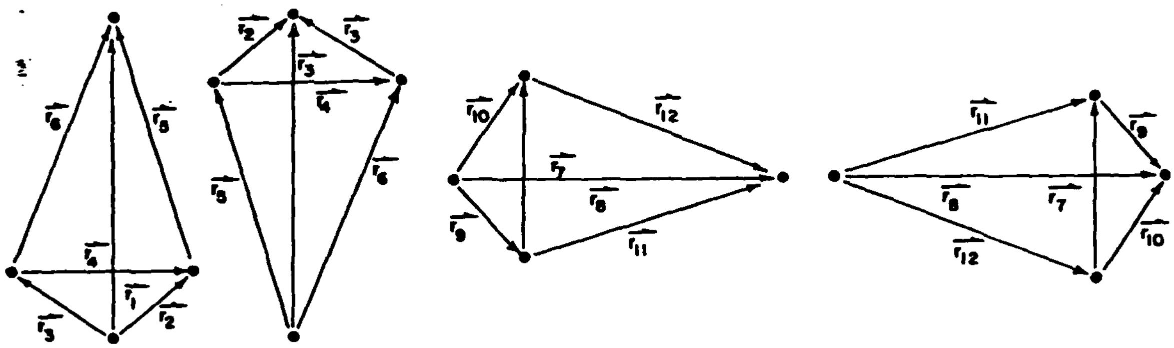
and 6-disk and the most general method are all very similar. They all use the same basic trick to generate texture pairs with identical second-order probabilities. Consequently, considering the 4-disk method, which is the simplest procedure of the three, is all that is needed to show how the discrimination could be performed on texture pairs generated by any of these methods.

Next, since the same basic trick is used in the generation of all the texture pairs created by the 4-disk method, albeit that the pattern geometry varies, we need only consider one example texture pair generated using this method. Further to make the analysis as simple and as understandable as possible we will use one of the simplest of the example texture pair which can be generated by the 4-disks method.

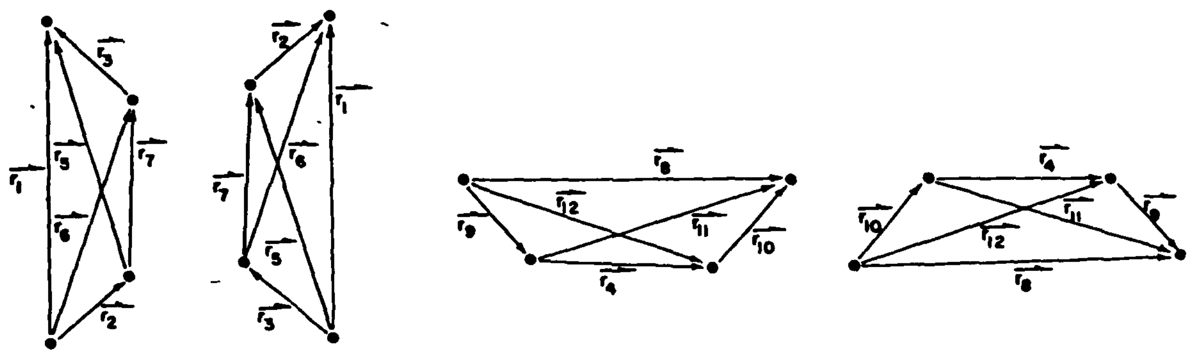
This texture pair is shown in Figure 14. Figure 15 shows the micro-patterns used to create the texture pair of Figure 14. Also shown in Figure 15 are labeled "dipoles" of each of the tiles. These "dipoles" are vectors which show the relative geometry of the patterns. One can show that these two textures have identical second-order probabilities by counting the number of times a particular labeled dipole occurs in each texture. This count is accomplished by merely counting the number of occurrences of that dipole in the tiles used to create the texture. It can be easily verified that each dipole occurs the same number of times in each texture.

What makes this example of the 4-disk method simple is that the geometry of the disks have been chosen so that only four angular rotations must be used to get the second-order probabilities of both patterns to be equal. These rotation angles are 0°, 90°, 180° and 270°.

A method for discriminating these two textures proceeds as follows. First the spatial gray level dependence matrices are computed globally for several d and θ values. Since the two textures have identically



(a)



(b)

Figure 15. The set of micropatterns used to generate the texture pair of Figure 14. (a) Is the set used to generate the texture on the left. (b) The set used to generate the texture on the right. Also shown are the dipoles of each of the micropatterns.

the same second-order probabilities the resulting corresponding spatial gray level dependence matrices would be approximately equal (remember there is an estimation error) for the two textures. Hence, the textures cannot be discriminated using this global analysis. However, the inertia feature computed from both textures can be used to estimate the size and shape of the period parallelogram unit pattern for each texture. Obviously the calculation yields the same size and shape unit pattern for both textures.

Appropriately centering this size parallelogram, in this case square region, allows the textures to be decomposed into a series of micropatterns. These patterns are shown in Figure 15.

The process continues by extracting the spatial gray level dependence matrices from each of the local square regions. It is these spatial gray level dependence matrices which will allow the discrimination. In particular it can be shown that the spatial gray level dependence matrices computed for $\delta = \vec{r}_1$, $\delta = \vec{r}_7$, $\delta = \vec{r}_8$ and $\delta = \vec{r}_4$ will allow perfect discrimination of the two textures.

The interesting point about the above example is that the global calculations indicate the scale on which the local calculations need be performed.

Next let us consider the even/odd textures. These textures are pure and hence theoretically one would desire perfect discrimination between the two. The even/odd textures as well as those generated by the Gagalowicz and the authors' synthesis procedure are very interesting in terms of the global/local analysis procedure.

The analysis of these patterns proceeds as above. The spatial gray level dependence matrices are computed globally for several d and θ values. Since these textures have identical second-order probabilities the resulting corresponding spatial gray level dependence matrices will be approximately equal. Consequently no discrimination on the global level is possible.

However, unlike the above an examination of the inertia feature computed globally seemingly gives no information about the scale on which the local analysis should proceed. The inertia feature is approximately constant with value ½. This is because all the second-order probabilities for the two textures have value ¼.

The apparent failure of the global anlaysis to provide any information concerning the scale of the local analysis will be considered later.

For now, let us concentrate on showing that the local analysis will allow the discrimination of the textures. To see this consider the patterns shown in Figure 16. These patterns represent all the 2 x 2 fundamental unit patterns of the even and odd textures. Also shown in the figure are the horizontal spatial gray level dependence matrices computed for these patterns. Note that the values of the spatial gray level dependence matrices computed from the 2 x 2 patterns making up the odd texture are different from values of the spatial gray level dependence matrices computed from the 2 x 2 patterns comprising the even texture. This difference could be used to discriminate them with 100% correct classification.

It should be noted that the value of the inertia feature computed from the odd texture pattern spatial gray level dependence matrices of Figure 16 is equal to ½ in all cases. While on the other hand the value of the inertia feature computed from the matrices of the even texture is never equal to ½. Consequently the discrimination rule for classifying these textures is that if the horizontal (d=1) inertia feature = ½ classify the sample as part of the odd texture otherwise call it part of the even texture.

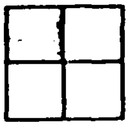
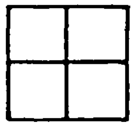
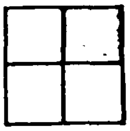
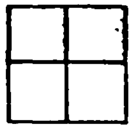
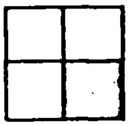
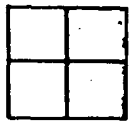
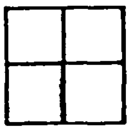
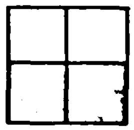
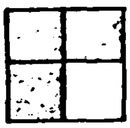
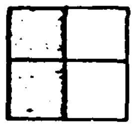
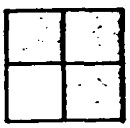
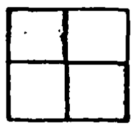
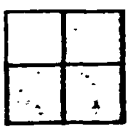
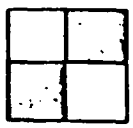
ODD TEXTURE		EVEN TEXTURE	
2 × 2 PATTERNS	$S(\delta), \delta = (1, 0^{\circ})$ MATRICES	2 × 2 PATTERNS	$S(\delta), \delta = (1, 0^{\circ})$ MATRICES
	$\begin{bmatrix} 0 & 1/2 \\ 0 & 1/2 \end{bmatrix}$		$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
	$\begin{bmatrix} 0 & 0 \\ 1/2 & 1/2 \end{bmatrix}$		$\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$
	$\begin{bmatrix} 0 & 0 \\ 1/2 & 1/2 \end{bmatrix}$		$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$
	$\begin{bmatrix} 0 & 1/2 \\ 0 & 1/2 \end{bmatrix}$		$\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$
	$\begin{bmatrix} 1/2 & 1/2 \\ 0 & 0 \end{bmatrix}$		$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$
	$\begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix}$		$\begin{bmatrix} 0 & 1/2 \\ 1/2 & 0 \end{bmatrix}$
	$\begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix}$		$\begin{bmatrix} 0 & 1/2 \\ 1/2 & 0 \end{bmatrix}$
	$\begin{bmatrix} 1/2 & 1/2 \\ 0 & 0 \end{bmatrix}$		$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Figure 16. The set of all possible 2 × 2 patterns for the even and odd textures and the corresponding spatial gray level dependence matrix, $S(\delta), \delta = (1, 0^{\circ})$ computed from each of these patterns.

Now that it has been established that a local analysis will allow the discrimination of the even and odd textures, let us return to the seeming failure of the global analysis to indicate what scale at which local analysis should be conducted. To address this question we examined the unit patterns of the even and odd textures which were of the following sizes, 2 x 2, 4 x 4, 8 x 8 and finally 12 x 12. In this analysis we wrote a computer program to generate every possible unit pattern of these various sizes. From each of the unit patterns for a particular size we extracted the horizontal (d=1) spatial gray level dependence matrix. For each possible size and each of the even and odd textures we created a histogram of the relative frequency of occurrence of each of the possible horizontal spatial gray level dependence matrices. The results of this experiment indicated that regardless of the scale used 2 x 2, 4 x 4, 8 x 8, or 12 x 12, the even and odd textures could still be discriminated with 100% accuracy based on the values of the horizontal spatial gray level dependence matrices. Further we found that the inertia feature still provided, regardless of scale, 2 x 2, 4 x 4, 8 x 8, or 12 x 12, a means for obtaining a 100% correct discrimination of the two textures. In particular, in each instance the same classification rule could be used to do the discrimination, if the horizontal (d=1) inertia feature equals ½ label the sample as part of the odd texture otherwise call it part of the even texture.

We did not consider scales bigger than 12 x 12 pixels only because of the computational time and memory involved in doing the calculations.

What these experiments seemingly indicate is that the scale on which the local analysis proceeds may not be all that important in this case. If this is true then the fact that the inertia feature is constant for all d and θ should not be upsetting. We will consider this problem further in the next texture pair analyzed.

As of this point we have shown either directly or by inference how each of the known counterexamples to Julesz conjecture which are members of the so called pure group could be discriminated. In each instance a 100% correct classification accuracy was obtained. We now consider the known counterexamples to the Julesz conjecture which are members of the impure group. In doing so we learn more about the nature of the global/local analysis. In particular, examine more examples of the phenomena when the inertia features is completely flat for all d and θ.

We begin our examination of the impure group by considering the texture pairs generated using the authors' synthesis method. The global analysis of these textures while not allowing discrimination does show that the two textures are both periodic in the $\theta = 0°$. For $\theta \neq 0°$ the inertia feature is completely flat or approximately flat and hence no information concerning the scale at which the local analysis should proceed is provided.

As we did above when a similar set of circumstances prevailed themselves, we will show first that a local analysis will allow the discrimination of these two textures. Figure 17 shows all the fundamental 2 x 4 patterns for each of the two textures. Note that the horizontal dimension of the pattern results from the detected periodicity and the measure of the period provided by the inertia measure. Also shown in the figure are the values of the vertical (d=1) spatial gray level dependence matrices computed from each of the fundamental 2 x 4 patterns. One should observe that the values of vertical (d=1) spatial gray level dependence matrices are the same for patterns 1, 2, 5, 6, 8, 11, 12, 15 and 16, in texture 2 and the

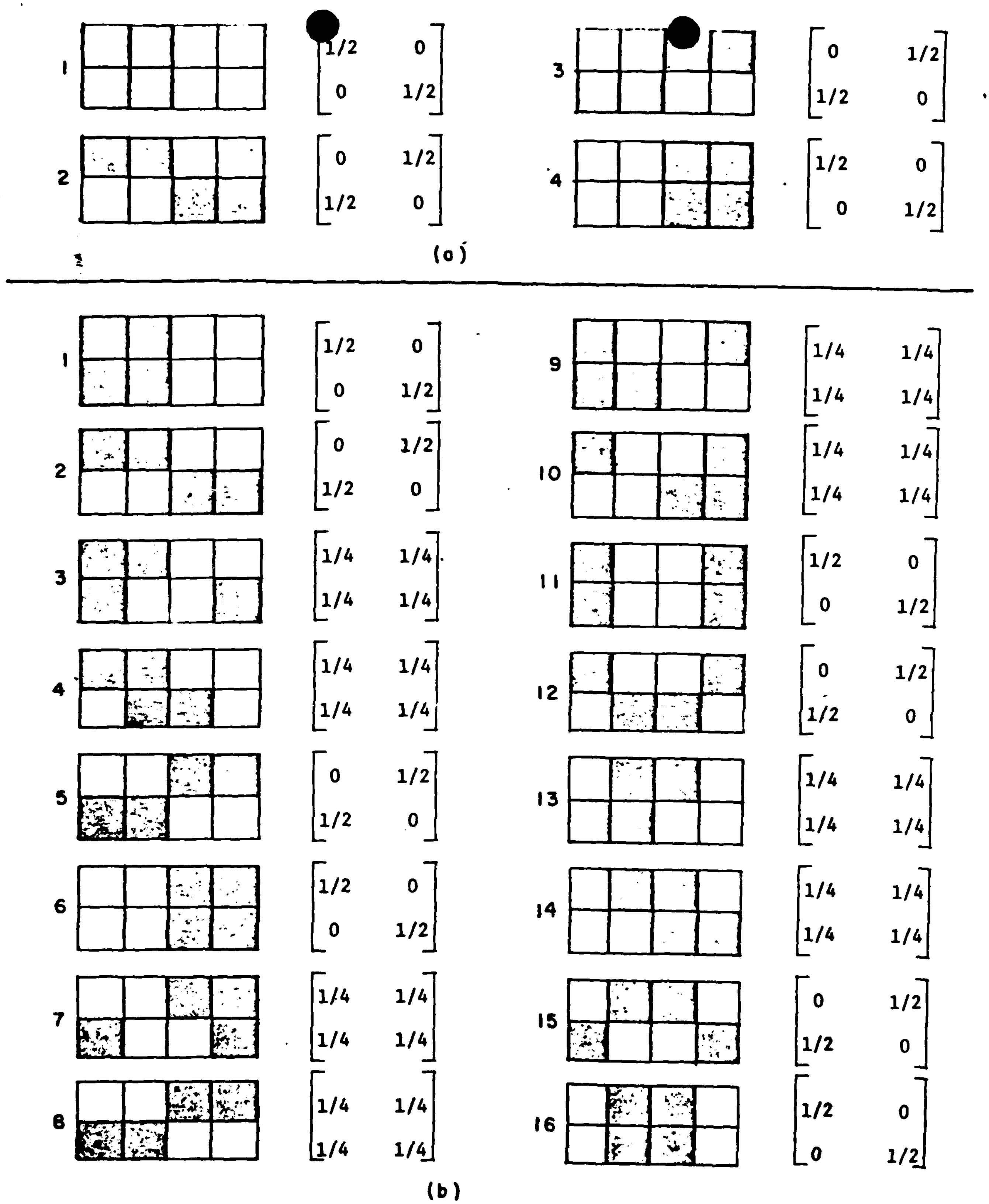


Figure 17. The set of all possible 2 × 4 patterns and the corresponding spatial gray level dependence matrix $S(\delta), \delta = (1, 90^{\circ})$. (a) The set of all possible 2 × 4 patterns created from Part a of Figure 12. (b) The set of all possible 2 × 4 patterns created from Part b of Figure 12.

four patterns and in texture 1. While this at first might seem a shortcoming, it is in actuality not a problem. Patterns 11, 12, 15 and 16, repsent translations or shifts of the patterns of texture 1. Such shifts could occur naturally as one goes from global to local analysis, i.e. through imprecision in placing the regions to be used in the local analysis.

Table 2 shows the probability of occurence of each possible vertical matrix for each of the two textures. An examination of these class conditional probabilities will show that classical pattern recognition methods will allow the discrimination of the two textures. In particular, texture 1 can be identified with 100% accuracy. On the other hand 50% of the texture 2 samples will be correctly labeled while the rest will be labeled texture 1.

Now that it has been shown that a local analysis will provide discrimination, we would like to further consider the ramifications of the inertia feature being constant at ½ for all d and $\theta \neq 0°$. To examine this problem we will consider variety of scales for conducting the local analysis and examine how these variations affect classification accuracies. The scales considered are 2 x 4, 4 x 4, 8 x 4, and 12 x 4.

The method of procedure was to generate every possible pattern of a particular size for each texture. For each pattern the vertical spatial gray level dependence matrices were extracted and a histogram of the probability of occurence of each matrix value was computed.

To see how this all works, let us consider only the vertical (d=1) spatial gray level dependence matrix. We extracted this matrix from all possible patterns of sizes 2 x 4, 4 x 4, 8 x 4, and 12 x 4. For each size considered we made a histogram of all the possible matrix values for each texture. The results indicated that the classification accuracy using just this one matrix did not vary with pattern size.

All possible $S(\delta), \delta = (o, 90^o)$	Probability of Occurrence in Texture on Left in Figure 12	Probability of Occurrence in Texture or Right on Figure 12
$\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$	.50	.25
$\begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}$	0.0	.50
$\begin{bmatrix} 0 & 1/2 \\ 1/2 & 0 \end{bmatrix}$	.50	.25

Table 2. The set of all possible vertical (d = 1) spatial gray level dependence matrices for the 2 × 4 patterns of Figure 17. Also given are the probabilities of occurrence of these matrix values in each of the two textures of Figure 13.

Next we performed the same type of experiment only we used two vertical matrices, d=1 and d=2, used in combination. In this instance the pattern sizes considered were 4 x 4, 8 x 4 and 12 x 4. Again it was observed that the classification accuracies obtainable using the two matrices did not vary with the size of the pattern from which they were extracted.

Figure 18 summarizes the set of experiments performed. The theoretical limit shown for the probability of error takes into account the impurity of the patterns.

An interesting observation can be made concerning the data presented in Figure 18. The data shown would seemingly indicate that if one wanted to achieve the best possible classification accuracies in discriminating these two textures one should conduct the local analysis on as large an area as possible and consider as many d and θ values as possible. Given this observation it is conceivable that the inertia measure being constant for all d and θ with value equal to ½ could imply that one should conduct the local analysis on as large a scale as is possible. The interesting point is that this interpretation is consistent with the procedures now used. To determine periodicity one looks for minimums in the inertia feature value. On textures where the inertia feature is perfectly flat the minimum occurs at infinity.

The last counterexample we have left to examine is the one generated using the Gagalowicz synthesis procedure. The global analysis of the two textures comprising this texture pair again returns only that the two textures cannot be discriminated globally. There is no information again concerning the scale at which the local analysis should proceed.

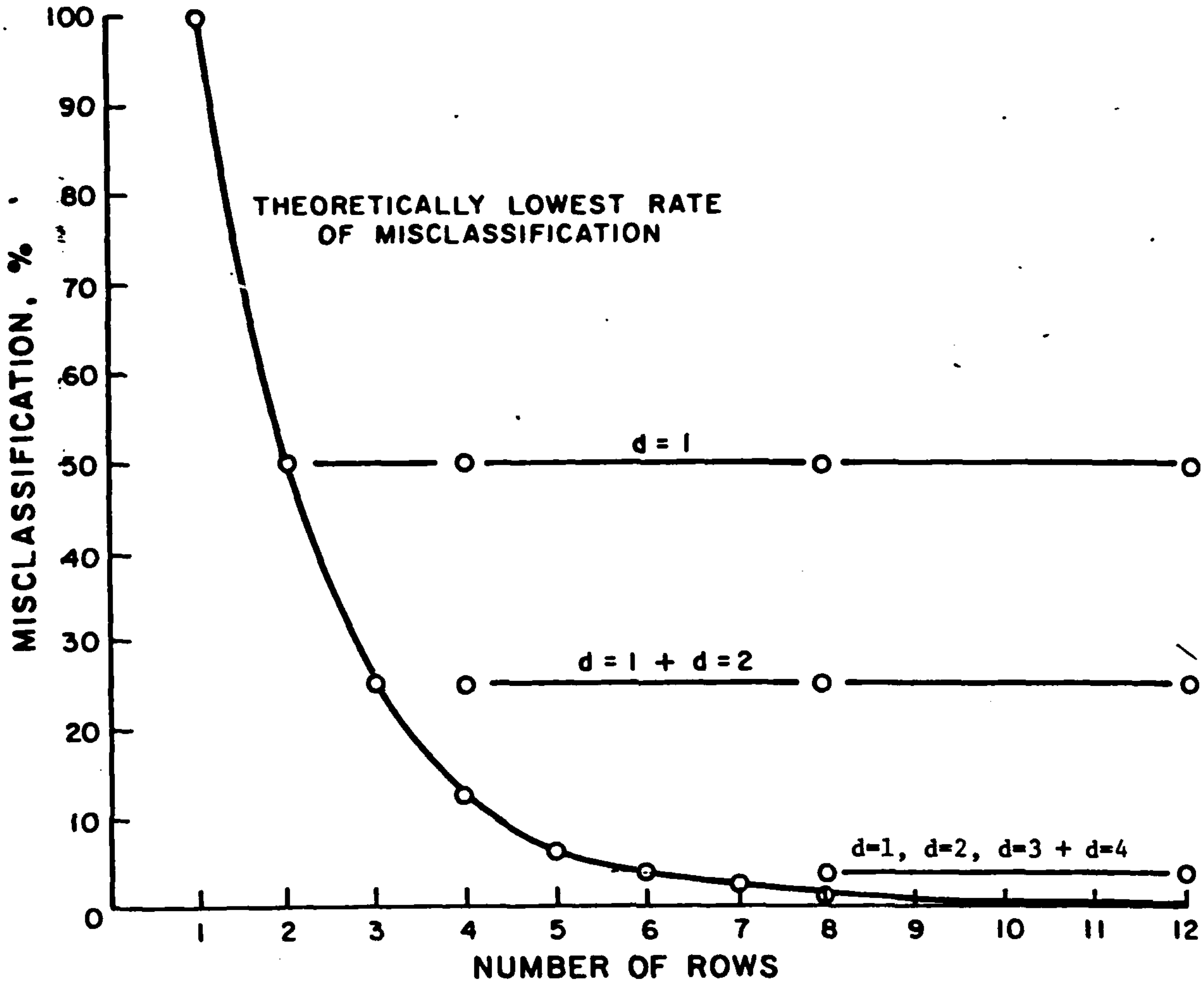


Figure 19. The theoretical versus actual rate of misclassification obtained for the texture on the left in Figure 13. The classification rates are shown as a function of the number of rows in the region from which the spatial gray level dependence matrices were computed. As can be seen when only one d value is considered, namely d = 1, an error rate of 50% is obtained for all n values used. When two d values are used, namely, d = 1 and d = 2, an error rate of 25% is obtained. Again this misclassification rate seems independent of the size of n. Finally notice that when three values of d, namely, d = 1, d = 2, d = 3 and d = 4 are used, a misclassification rate of 3.61% is obtained. Again this error rate seems independent of the values of n.

A local analysis will allow the discrimination. In particular the local analysis can be used to isolate the inhomogeneities which allow the visual discrimination of these textures.

Unfortunately at this time we do not know whether for these textures the discrimination accuracy is independent of the region size considered. The difficulty in making this determination stems from the very large number of possible patterns that need to be considered. This number gets so large so quickly that it is difficult to analyze enough cases to arrive at any meaningful conclusions.

It should be pointed out that the global/local analysis procedure described can also be used to discriminate the texture pairs in Figures 6 and 7 . Note that these textures are not spontaneously discriminable to humans.

The above remarks lead us to Assumption 3.

Assumption 3: If two textures are visually distinct then there exist a region size such that the distribution of estimated second-order probability matrices computed using this region size will be different for the two textures.

C. *On Developing New Features for the SSA*

In the last section we showed that the known counterexamples to the Julesz conjecture could be discriminated using the SGLDM. Further, it was shown that the discrimination process used was not an artifically conceived one but rather one that follows the natural operating procedure of the SSA.

The fact that the discrimination of these textures could be accomplished in this way was most encouraging to the authors. It further verified the methodologies being employed. Consequently what remains is to continue the development of the SSA. We have seen that there are example texture pairs which cannot be discriminated using only the inertia feature, the only feature currently in the SSA Level 0 feature set. What remains then is defining more Level 0 features, features which when used as data for Level 1 analysis will allow inferences to be made about visually perceivable qualities of the patterns being examined.

Obviously the structure of the SSA is such that its overall capabilities depend completely on having high quality Level 0 features. Since the Level 0 features are critical to total system performance one must be very careful in defining them. One needs a formal process which relies heavily upon experimentation and/or theoretical development.

Since our objective is to have features which measure visually perceivable characteristics of patterns and since very little is known about the mathematical structure of the visual system, a theoretical development does not seem possible. It might be possible to do a theoretical development if one would restrict the class of textures considered to be those which can be represented by some well understood set of random fields. However, for our purposes this restriction would be self defeating. We are interested in developing methods that can be used on any texture analysis problem.

Consequently experimental approaches seem to be the way to proceed. Given this fact what needs to be developed then are a set of techniques which will allow us to hold certain parameters constant and allow others to vary. Within the context of the feature development problem this means that one must have methodologies available which will allow the generation of texture pairs which have some feature values identical for both textures and some one or more feature values different.

In the following we present our progress in developing a framework for the feature definition problem. In particular we will state our design objectives and the associated design steps. Further we will give some mathematical techniqes which can be used in the parametric studies.

We begin by stating our design guidelines for feature development. These guidelines do not guarantee uniqueness in the feature development process, i.e., two investigators using these guidelines might arrive at two completely different feature sets both of which meet all the design criteria. However, in most engineering design problems the design objectives seldom completely specify the resulting system. These guidelines embody, many of the desirable traits one would like in a feature set.

The design guidelines can be briefly stated as follows.

DG1. Each feature in the feature set should "measure" some visually perceivable quality of a texture pattern.

DG2. The features in the feature set should be "independent."

DG3. Given any visually distinct pair of textures there should be at least one feature in the feature set such that the value of this feature is different for the two textures comprising the pair.

To avoid any ambiguity, two words, namely "measure" and "independent", need be precisely defined. For a feature to "measure" a visually perceivable quality requires that the feature value be a monotonic strictly increasing function of that quality. That is as the visual quality becomes more prominent as the value of the feature increases.

The meaning of the word "independent" can best be described by example. Suppose that one has a feature set comprised of three features, F_1, F_2 and F_3. These features are said to be independent if:

i) there exists a visually distinct texture pair such that the expected values of the features F_2 and F_3 are the same for both textures in the texture pair but the expected value of F_1 is different for the two textures;

ii) there exists a visually distinct texture pair such that the values of F_1 and F_3 are the same for both these textures but the expected value of F_2 is different;

iii) there exists a third visually distinct texture pair such that the expected values of features F_1 and F_2 are the same for both these textures but the expected value of feature F_3 is different for these two textures.

The extension of the above example to n features is straightforward. DG2 substantively requires that each feature be uncorrelated.

In these design guidelines we require that the features measure visually perceivable qualities of patterns. From our discussions concerning the SSA we automatically know then that these deisgns apply to the Level 1 features since Level 0 cannot directly measure visual qualities. However, the challenge here is to define the Level 0 features in such a way that it can clearly be substantiated that a Level 1 feature can be defined which will measure a visual quality. This was precisely the technique used in [1] when we showed that the inertia feature could be used to detect periodicity.

The above design guidelines require certain steps be performed in the design process. If the investigator is iteratively defining features as would normally be the case, then the following steps are required:

DS1. demonstrate the need for an additonal feature in the feature set;

DS2. abstract the visual quality which need be measured and design a candidate feature to measure this quality;

DS3. establish that the candidate feature does monotomically measure this visual quality;

DS4. establish that the candidate feature is "independent" of the other features already defined.

If one can perform all these steps then according to the design guidelines the candidate feature may be added to the feature set.

Of course, design guidelies are of utility only if the design steps associated with these guidelines can, in fact, be reasonably performed. Consequently, a careful examination of the design steps associated with the stated guidelines are in order.

Let us consider DS1. Suppose one has already defined n features, $y_1, y_2, \ldots, y_n$. To demonstrate that a new feature need be added to the feature set, one must find a visually distinct texture pair which has identical values for all the $y_1, y_2, \ldots, y_n$. That is, a visually distinct texture pair must be found which cannot be discriminated by any of the existing features comprising the feature set. To find such a texture pair requires one to be able to create texture pairs which have controllable feature values. Obviously too, it would be helpful if there were some theory one could employ which would restrict the dimensionality of the space that must be examined to find such a visually distinct texture.

DS2 requires one to abstract a visual quality that is not being measured by any of the existing features and then to define a candidate feature to measure this quality. Of all the design steps this is the one which is most heavily dependent on the creativity of the investigator. Nonetheless, even here, certain basic tools are useful. First, one must have some mechanism for creating textures which have controllable feature values. That is, one only wants to look at texture pairs which have all the previous feature values held constant. Another tool that would be useful would be to have a mechanism by which to characterize all textures which have identical feature values for the n previously defined features. If these tools were available, the difficulty of the problem set forth in the design steps could be greatly reduced.

DS3 requires one to validate that the candidate feature does in-fact monotonically measure the visual quality abstracted. The best way to assure this is to be able to generate sets of textures such that all textures in a set have the same values for each previously defined feature but which have steadily increasing values for the candidate feature. Examining several such sets of textures should indicate whether the candidate feature is a monotonic function of the abstracted visual quality. The procedures used in DS3 are also applicable to DS4. DS4 merely requires that one be able to demonstrate a visually distinct texture pair such that both textures comprising this pair have identical values for all the previously defined features but that they have different values for the candidate feature.

From the above it should be clear that what is needed is a methodology for synthesizing textures which have controllable feature values. In what follows we will describe such a procedure. This procedure can be used in the development of Level 0 features for the SSA.

Basically the mathematical techniques employed to provide this synthesis capability have two basic steps. The first step is to find two second-order probability matrices which have identical feature values for any group of features, say F_1, F_2 and F_3, and a maximum difference in any selected feature, say F_4. The second part of the process uses these two second-order probability matrices to create a texture pair such that the values of F_1, F_2 and F_3 are identical for all d and θ and the value of F_4 is different for the two textures.

In formulating this synthesis procedure an assumption is made concerning the general form of the Level 0 features. While this assumption is not absolutely necessary in that other synthesis methods could be derived without it, the assumption greatly simplifies the calculations required.

Assumption 4: All Level 0 features will be linear with respect to the elements $s(i,j,\delta)$ of the matrix $S(\delta)$. That is each Level 0 feature is of the form

$$\sum_i \sum_j a_{ij}\, s(i,j,\delta).$$

While at first this assumption might seem arbitrary and very restrictive, it is important to note that it is not a restrictive as it might seem. First, it can be argued that the important texture information in a spatial gray level dependence matrix is determined by distribution of the probabilities within the matrix, i.e., the distribution with respect to i,j location. Classically the way this type of distribution has been measured is by using various moment funtions. The interesting point is that any moment defined on the spatial gray level dependence matrix is a linear function of the elements $s(i,j,\delta)$. Consider for example the inertia feature; this feature is a type of moment and is defined by

$$\sum_i \sum_j (i - j)^2 \, s(i,j,\delta).$$

Note that this expression is linear with respect to the elements $s(i,j,\delta)$.

Further it is important to note that not only are all classical types of moment function linear functions of the elements $s(i,j,\delta)$ but that there are many other functions which are linear with respect to the elements $s(i,j,\delta)$ which are not moments. Consequently the linearity assumption should not seem all that restrictive.

Using the linearity assumption the equations defining two spatial gray level dependence matrices, $S_1 = [s_1(i,j)]$ and $S_2 = [s_2(i,j)]$, which have n feature values equal and one feature value being maximally different are given by the following:

$$\Sigma\ \Sigma[c_{ij}\ s_1(i,j) - c_{ij}\ s_2(i,j)] = 0 \qquad (1)$$

$$\left.\begin{array}{l}
\Sigma\ \Sigma[a^1_{ij}\ s_1(i,j) - a^1_{ij}\ s_2(i,j)] = 0 \\
\Sigma\ \Sigma[a^2_{ij}\ s_1(i,j) - a^2_{ij}\ s_2(i,j)] = 0 \\
\vdots \\
\Sigma\ \Sigma[a^n_{ij}\ s_1(i,j) - a^n_{ij}\ s_2(i,j)] = 0
\end{array}\right\} \qquad (2)$$

$$\left.\begin{array}{l}\Sigma\,\Sigma\, s_1(i,j) = 1 \\ \Sigma\,\Sigma\, s_2(i,j) = 1 \\ \Sigma\, s_1(i,j) = P(j) \text{ for all } j \\ \Sigma\, s_1(i,j) = P(i) \text{ for all } i \\ \Sigma\, s_2(i,j) = P(j) \text{ for all } j \\ \Sigma\, s_2(i,j) = P(i) \text{ for all } i \\ s_1(i,j) \geq 0 \text{ for all } i,j \\ s_2(i,j) \geq 0 \text{ for all } i,j \end{array}\right\}$$

where c_{ij} are the coefficients defining the feature whose value is to be maximally different and a^1_{ij}, a^2_{ij}, ..., a^n_{ij} represent the coefficients defining the n features which are to have identical values. Here the a^1_{ij} are the coefficients defining the first feature, a^2_{ij} are the coefficients defining the second feature, etc. Finally the P(i) represents the first-order probability of the gray levels for the two patterns. Here we let $P(i) = 1/L$ for $0 \leq i \leq L - 1$ where L is the number of gray levels to appear in the two synthesized textures.

It should be noted that the notation used to represent the matrices $S_1 = [s_1(i,j)]$ and $S_2 = [s_2(i,j)]$ is different than that used to represent typical spatial gray level dependence matrices. This notational difference is employed to avoid confusion since these matrices are only used to generate the appropriate textures.

These equations define in effect a linear programming problem. Eq. 1 is the cost functional, Eq. 2 and Eqs. 3 being the set of linear constraints. This set of equations can be solved using the Simplex algorithm. The resulting values of $s_1(i,j)$ and $s_2(i,j)$ define the matrices S_1 and S_2.

The next step in the synthesis procedure is to generate two texture using S_1 and S_2 such that the resulting texture pair have identical values for the n features for all d and θ and different values for the features selected to be maximally different. As it turns out this too becomes a linear programming problem.

The procedure employed to actually generate the textures is a variant of the Gagalowicz synthesis procedure. This variation results from the necessity to

1. consider second-order probabilities different from $1/L^2$ where L is the number of gray levels;

2. to make the synthesis method require as little input as possible and hence hopefully making it easier to use.

In this variant only horizontally adjacent points can be used. Thus the geometry is fixed. The input parameters are L, K (defined in the discussion of the Gagalowicz synthesis procedure) and the matrices S_1, and S_2. Two textures which result have all the desired properties.

Unfortunately, the only difficulties with this procedure are

1. the complexity of the problem grows rapidly as the values of K and L increase;

2. only a limited family of textures can be generated in this fashion.

Nonetheless, this synthesis method will begin to allow us to do feature development on a sound albeit limited testing basis.

Using the above procedures we have defined a new Level 0 feature for the SSA. It is call the cluster shade feature. To understand what this feature does consider the textures shown in Figure 19. These two textures cannot be discriminated by the inertia feature. They can however be discriminated by the cluster shade feature. The cluster shade feature gives information concerning the size and shade of clusters. In the textures shown in Figure 19 this feature allows the discrimination

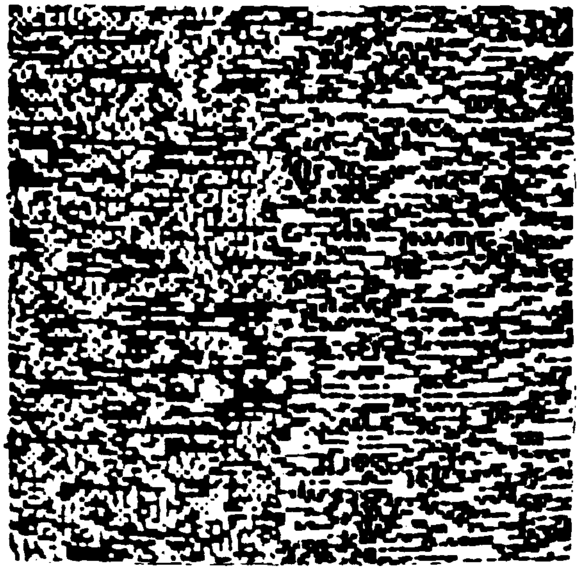


Figure 19. Two visually distinct textures which cannot be discriminated by the inertia feature. These two textures, however, can be discriminated by the cluster shade feature.

of these textures by sensing the fact that in one texture black clusters are prominent and in the other texture white clusters are prominent.

As of this writing we are just beginning our search for other possible features.

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IV. PUBLICATIONS

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In addition a paper is currently being prepared which will describe most of the feature development work that has been completed during this last year.

V. List of Personnel

Charles A. Harlow	Professor
Richard W. Conners	Professor
Ramon Emi Vasquez Espinosa	Graduate Student Ph.D. Candidate
Don Dirosa	Graduate Student Master Candidate
King Yao Lin	Graduate Student Ph.D. Candidate
John Bourg	Undergraduate Student

VI. INTERACTIONS

During the second year of the project, substantial efforts have been made to interact with U.S. government laboratories interested in image processing and related research. Specific contacts are as follows:

a. RADC, Griffiss AFB
Rome, N.Y.
D. Bush

b. DMMAC
St. Louis, Missouri
Dan W. Rusco

c. U.S. Army Corps of Engineers
J. Stoll
V. Lagarde
Environmental Laboratory
Vicksburg, MS

d. NASA
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e. U.S. Army Missile Laboratory
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